

Postbuckling Behavior of Heated Skin Panels

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The behavior of heated skin panels when subjected to pressure variation and flow from engine wake is strongly influenced by the buckling mode. The purpose of this note is to present analysis to predict when the first mode (which tends to stabilize the skin panel) is replaced by the second mode which may lead to failure by a rippling phenomenon.

Nomenclature

- a = length of plate
- b = width of plate
- t = thickness of plate
- w = buckle deflection
- A = frame area
- B = defined in Eq (1)
- E = modulus of elasticity
- R = radius of curvature
- ΔT = temperature difference between skin and frame
- α = coefficient of thermal expansion
- δ = axial shortening in buckling mode
- ν = Poisson's ratio
- σ = stress

Subscripts

- cr = critical
- f = frame
- o = center
- x = x direction
- y = y direction

THE first mode is characterized by a general bowing of the panel in one direction as is shown in Fig 1a. The second mode is characterized by multiple half-waves in the longer dimension of the panel (Fig 1b). Adding or subtracting a half-wave results in positions of near equilibrium, and this is believed to result in high-stress amplitude for low-level input and to account for the rippling appearance.

Violent rippling recently was observed at Lockheed-Georgia Company on a test to determine the thermal environment for the C-141 flaps. The dummy flap panels were constructed with light gage skin and a minimum of stiffeners to facilitate rapid attainment of the thermal equilibrium conditions. The panels, which were buckled in the second mode, failed before temperature measurements could be made.

Examination of the literature adds credence to the contention that panels subjected to sonic fatigue should be designed so that they do not buckle in the second mode. Panels buckled in the first mode are known to have passed development tests for similar environment (B-58 elevons). From panel flutter literature, buckling in the first mode may tend to stabilize the panel,¹ and buckling in some short wave modes leads to unstable oscillations for low values of the speed parameter.²

The buckling behavior of a panel subjected to biaxial thermal stress is similar to that of panels subjected to combined normal pressure and axial load in Ref 3. The deflection in the first mode corresponds to the deflection due to pressure, and the buckling in the second mode may be delayed until the stress is a multiple of the critical value for a flat panel.

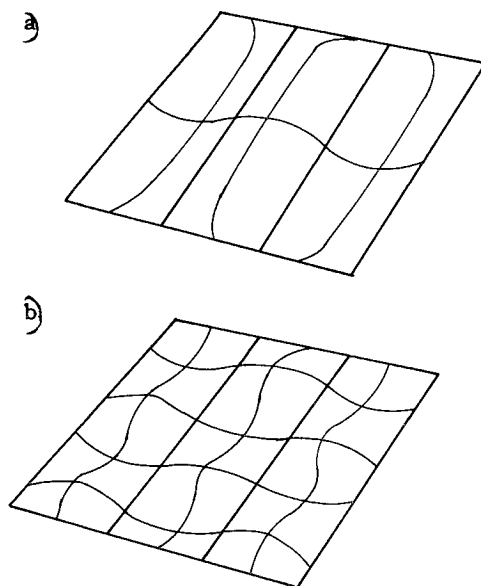


Fig 1 Buckling modes of skin panels; a) first mode, b) second mode

Analysis

The skin panel is assumed to be long, simply supported, elastic, restrained by cool substructure, at uniform temperature, and subjected to increasing temperature differential.

A simple criterion is utilized to predict buckling, but it approximates the results of the large deflection analysis in Ref 3. Dimension and coordinates are indicated in Fig 2.

Two quantities are arbitrarily defined to simplify the expressions for thermal stress:

$$B_x = 1 + Eat/E_f A_x \quad (1)$$

$$B_y = 1 + Ebt/E_f A_y$$

The ratio of the thermal stresses in the x and y direction prior to buckling is

$$\sigma_x/\sigma_y = (B_y + \nu)/(B_x + \nu) \quad (2)$$

Combined stresses for buckling of long flat plate are shown on Fig 3, together with the zones for initial buckling in the first and the second modes.[†] The simply supported plate

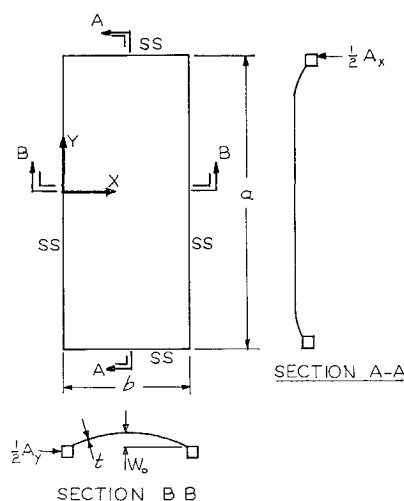


Fig 2 Typical skin panel

[†] Estimated value for clamped plates are also shown on Fig 3. Note that rotational restraint may lead to initial buckling in the second mode as in the case of the clamped plate for $\sigma_x = \sigma_y$.

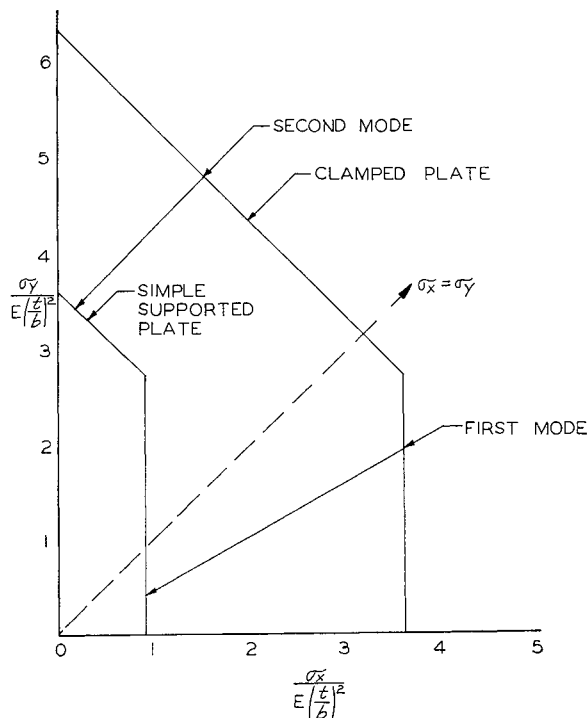


Fig 3 Buckling of flat plates under combined stress

buckles into the first mode when the stress in the x direction, which is larger than $\frac{1}{3}$ the stress in the y direction, reaches the following value:

$$\sigma_x = 0.9E(t/b)^2 \quad (3)$$

and the deflection form is

$$w = w_0 \sin(\pi x/b) \quad (4)$$

so that the unit axial shortening in the buckle mode is

$$\frac{\delta}{b} = \left(\frac{1}{2b}\right) \int_0^b \left(\frac{dw}{dx}\right)^2 dx = \frac{w_0^2 \pi^2}{4b^2} \quad (5)$$

The condition of compatibility in the x direction gives

$$\alpha \Delta T = \sigma_x B_x / E - \nu \sigma_y / E + \delta / b \quad (6)$$

Substituting Eqs (3) and (5) in Eq (6) and solving for w_0 yields

$$w_0 = (2b/\pi) [\alpha \Delta T - 0.9(t/b)^2 B_x + \nu \sigma_y / E]^{1/2} \quad (7)$$

The critical stress condition for the curved plate is calculated by utilizing the following criterion:

$$(\sigma_x + \sigma_y)_r = 3.6E(t/b)^2 + 0.3Et/R \quad (8)$$

which was arrived at by adding the buckling stress of the cylinder to that of a square plate. The radius R in the previous equation is taken as the constant radius which gives the same deflection as the sine wave,

$$R = b^2/8w_0 \quad (9)$$

The equation for compatibility in the y direction reduces to

$$\sigma_y / E = (1/B_y) [0.9\nu(t/b)^2 + \alpha \Delta T] \quad (10)$$

Substituting Eqs (7, 9, and 10) in Eq (8) and setting $\nu = 0.30$ yields

$$\alpha \Delta T (b/t)^2 = 1.17B_y^2 + 3.05B_y - 0.27 \pm [1.36B_y^4 + 7.12B_y^3 + 1.38B_y^2 - 2.10B_x B_y^2]^{1/2} \quad (11)$$

As can be seen from Eq (11), the value of the parameter $\alpha \Delta T (b/t)^2$ at which the second mode occurs depends, in a rather involved way, on the values of the in-plane restraint param-

Table 1 Values of $\alpha \Delta T (b/t)^2$, for appearance of the second mode

| | | B_x | | |
|-------|---|-------|------|------|
| | | 1 | 2 | 3 |
| B_y | 1 | 6.74 | 6.33 | 5.84 |
| | 2 | 19.2 | 18.7 | 18.2 |
| | 3 | 36.6 | 36.1 | 35.5 |

eters B_x and B_y . A set of values of $\alpha \Delta T (b/t)^2$ for various restraints, including completely restrained $B_x = B_y = 1$, and lightly framed panels $B_x = B_y = 3$, is given in Table 1.

Concluding Remarks

All the values of $\alpha \Delta T (b/t)^2$ are significantly higher than the value 2.43, which is the lowest value corresponding to initial buckling in the second mode with complete inplane restraint of the heated panel in the y direction. The value of B_x is of minor importance as long as the panel buckles initially in the first mode. In the application of the preceding results it should be noted that the assumptions of simple support and uniform compression may lead to overoptimism where these conditions are not closely approximated.

References

- 1 Fung, Y. C., "Some recent contributions to panel flutter research," AIAA J 1, 898-909 (1963).
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- 3 Levy, S., Goldenberg, D., and Zebratosky, G., "Simply supported long rectangular plate under combined axial load and normal pressure," NACA TN 949 (October 1944).

Detonation of Hydrogen-Oxygen at Low Temperature and High Pressure

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THE detonation velocity of hydrogen-oxygen mixtures is of fundamental interest in the study of combustion instability in conventional rocket motors as well as in the research on unconventional rocket motors utilizing detonative combustion.¹ This note presents the experimentally obtained detonation velocity of gaseous hydrogen-oxygen mixtures at initial temperatures from room temperature to the vicinity of the oxygen vapor saturation point ($\sim 110^\circ\text{K}$) and initial pressures of 1-15 atm. Stoichiometric and hydrogen-rich mixtures were considered of primary interest. The previously existing experimental data (e.g., Refs. 2 and 3) thus have been extended.

The tests were conducted with a stainless steel tube, 0.25-in. i.d., 0.50-in. o.d., 20 ft long and coiled in a 10-in. diam. The curvature of the tube was shown to have a negligible effect on the detonation velocity. The velocity of the

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